MPZ 3231 - Assignment 2

(1) (i) Solve the following differential equations

(a) $y' = \frac{x y^3}{\sqrt{1+x^2}}$

(b) $y' = e^{y-t}\sec(y)(1+t^2)$

(c) $2xy^2 + 4 = 2(3 - x^2 y)y'$

(d) $\frac{2ty}{t^2+1} - 2t - \left(2 - ln(t^2+1)\right)y' = 0$

(e) $2(x+2y)\,dx + (y-x)\,dy = 0$

(f) $(x^2 - y^2)\,dx + xy\,dy = 0$

(ii) Determine whether the given differential equations are homogeneous and if so, solve them.

(a) $y' = \frac{y}{x+\sqrt{xy}}$

(b) $y' = \frac{x^4+3x^2y^2+y^4}{x^3 y}$

(iii) Solve the given differential equations or initial value problems.

(a) $dx - \frac{1}{y^2-6y+13}\,dy = 0$

(b) $(x^2+1)\,dx + \frac{1}{y}\,dy = 0 \;; y(-1) = 1$

(2) (i) Solve the following Initial Value Problem (IVP) using integrating factor method

(a) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1 \quad ; y\left(\frac{\pi}{4}\right) = 3\sqrt{2} \quad ; \; 0 \le x \le \frac{\pi}{2}$

(b) $ty' - 2y = t^5 \sin(2t) - t^3 + 4t^4 \quad ; y(\pi) = \frac{3}{2}\pi^4$

(ii) Solve the following differential equations

(a) $y^{(4)} + 2y^{(3)} + 3y^{(2)} + 2y^{(1)} + y = 0$

(b) $y^{(5)} - 5y^{(4)} - 50y^{(3)} + 250y^{(2)} + 625y^{(1)} - 3125y = 0$

(c) $y^{(6)} - 5y^{(4)} + 16y^{(3)} + 36y^{(2)} - 16y^{(1)} - 32y = 0$

(3) (i) Use the Bisection method to find the roots of the folowing equation

(a) $f(x) = x^3 + 2x^2 - 3x - 1$

(b) Find a root of the following, equations correct to three decimal places, using the Bisection method.

$3x - e^x = 0$

(c) Using Bisection method finds the negative root of $x^3 - 4x + 9 = 0$, correct to three decimal places.

(ii) Using Newton- Raphson method, find a root correct to three decimal places of the following:

(a) $e^x \sin x = 1$ (b) $x\,[1 - log_e x] = 0.5$ (c) $x \sin x + \cos x = 0\ near\ x = \pi$

(iii) $\int_0^5 \frac{dx}{4x+5}$

(a) Evaluate by taking 10 equal parts and using Simpson's one third rule.

(b) $x = \int_8^{30} \left(2000 \ln \left[\frac{140\,000}{140\,000 - 2100\,t}\right] - 9.8\,t\right) dt$

Evaluate by taking 4 equal parts and using Simpson's one third rule.

(4) (i) Estimate the population in 1895 and 1925 from the following statistics:

year x	1891	1901	1911	1921	1931
population y	46	66	81	93	101

(ii) (a) Given that $log_{10} 300 = 2.4771$, $log_{10} 304 = 2.4829$, $log_{10} 305 = 2.4843$ and $log_{10} 307 = 2.4871$, find by using Lagrange's formula, the value of $log_{10} 310$

(b) Given: $u_1 = 22, u_2 = 30, u_4 = 82, u_7 = 106$ and $u_8 = 206$. Find u_6 using Lagrange's interpolation formula.

(5) (a) Use the Jacobi method to approximate the solution of the following system of linear equations. Continue the iteartions until two successive approximations are identical when rounded to three significant digits.starting solution (0,0,0)

$5x_1 - 2x_2 + 3x_3 = -1$

$-3x_1 + 9x_2 + x_3 = 2$

$2x_1 - x_2 - 7x_3 = 3$

(b) Use the Gauss-Seidel iteartion method to approximate the solution to the system of equations given in above ((5) (a)).

(c) Solve the following system of equations using Gausssian elimination with partial pivoting.

$2x_1 + x_2 + x_3 = 5$

$4x_1 - 6x_2 = -2$

$-2x_1 + 7x_2 + 2x_3 = 9$